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Use of the Implicit-Finite-Difference Method to Implement the Parabolic Equation Model

C. H. Shellman

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NAVAL OCEAN SYSTEMS CENTER
San Diego, California 92152-5000

J. D. FONTANA, CAPT, USN
Commander

H. R. TALKINGTON, Acting
Technical Director

ADMINISTRATIVE INFORMATION

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J. A. Ferguson, Head
Ionospheric Branch

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J. H. Richter, Head
Ocean and Atmospheric
Sciences Division

SUMMARY

PROBLEM

Investigate alternate means to the split-step algorithm for determining coverage in the troposphere at gigahertz (GHz) frequencies.

RESULTS

The theory of the implicit-finite-difference method is summarized and the boundary conditions described. Examples indicate that, for sufficiently high signal levels, the IFD method is adequate at 9.6 GHz. However, the method did not give correct results beyond 35 km for the standard-atmosphere case in which signal levels are very low.

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I. INTRODUCTION

This report describes an effort to investigate alternate means to the split-step algorithm for determining radar coverage in the troposphere at gigahertz (GHz) frequencies. It concerns mostly the results of testing the implicit-finite-difference (IFD) algorithm described by Lee, Botseas, and Papadakis [1981]. Both methods are solutions to the parabolic wave equation (PE), for which reflections are neglected.

Lee, et al. [1981], give an analysis of truncation error for the implicit-finite-difference method and conclude that it is consistent, is stable, and converges. S. T. McDaniel [1975] concluded that the finite-difference method is useful only at low frequencies. However, results were obtained by this method at 9.6 GHz, except for one case in which signal levels are very low.

Section II of this report gives a summary of the theory of the implicit-finite-difference method. Section III summarizes the starting conditions used at the beginning of the integration of the parabolic equation. In section IV, three examples are given of use of the IFD method.

The characteristics of the tropospheric models are given as modified refractivity, M, versus height where $M = (n-1) \times 10^6 + 0.157\ z$, n is the index of refraction, and z is the height in meters. Results are given in terms of path loss versus distance.

II. THEORY OF THE IMPLICIT-FINITE-DIFFERENCE METHOD

The parabolic equation is of the form

$$\begin{aligned} \partial u/\partial r &= a(k_o, r, z)\ u + b(k_o)\ \partial^2 u/\partial z^2 \\ &= (a + bD^2)\ u = Lu \end{aligned} \tag{1}$$

where

$$a = (ik_o/2)[n^2(r,z) - 1]$$
$$b = i/2k_o$$
$$D = \partial/\partial z$$

and where k_o is the reference wave number, n is the index of refraction, r is range, and z is altitude. This can be expressed in the symmetrical form

$$e^{-1/2kL}\ u_m^{n+1} = e^{1/2kL}\ u_m^n \tag{2}$$

where m is an index with regard to mesh points in the vertical direction and n is an index with regard to mesh points in the horizontal direction. Claerbout [1970] indicates that use of a symmetrical form of solution results in considerably more accuracy.

To first order in the exponentials, equation 2 becomes

$$[1 - k(a_m^{n+1} + b_m^{n+1}D^2)/2]u_m^{n+1} = [1 + k(a_m^n + b_m^n D^2)/2]u_m^n. \tag{3}$$

This approximation inherently limits the mesh size. Nevertheless, in two ducting cases at 9.6 GHz, the method gave accurate results with reasonably large mesh size.

Following Lee, et al. [1981], equation 3 may be written in matrix form as

$$
\begin{bmatrix}
X_1 & -\frac{1}{2}\beta_1^{n+1} & 0 & \cdots & 0 & 0 & 0 \\
-\frac{1}{2}\beta_2^{n+1} & X_2 & -\frac{1}{2}\beta_2^{n+1} & \cdots & 0 & 0 & 0 \\
 & & & \vdots & & & \\
0 & 0 & 0 & \cdots & -\frac{1}{2}\beta_{m-1}^{n+1} & X_{m-1} & -\frac{1}{2}\beta_{m-1}^{n+1} \\
0 & 0 & 0 & \cdots & 0 & -\frac{1}{2}\beta_m^{n+1} & X_m
\end{bmatrix}
\begin{bmatrix}
u_1^{n+1} \\ u_2^{n+1} \\ \vdots \\ u_{m-1}^{n+1} \\ u_m^{n+1}
\end{bmatrix}
$$

$$
=
\begin{bmatrix}
Y_1 & \frac{1}{2}\beta_1^{n} & 0 & \cdots & 0 & 0 & 0 \\
\frac{1}{2}\beta_2^{n} & Y_2 & \frac{1}{2}\beta_2^{n} & \cdots & 0 & 0 & 0 \\
 & & & \vdots & & & \\
0 & 0 & 0 & \cdots & \frac{1}{2}\beta_{m-1}^{n} & Y_{m-1} & \frac{1}{2}\beta_{m-1}^{n} \\
0 & 0 & 0 & \cdots & 0 & \frac{1}{2}\beta_m^{n} & Y_m
\end{bmatrix}
\begin{bmatrix}
u_1^{n} \\ u_2^{n} \\ \vdots \\ u_{m-1}^{n} \\ u_m^{n}
\end{bmatrix}
$$

$$
+ \begin{bmatrix} \frac{1}{2}\beta_1^{n+1} & u_o^{n+1} \\ & 0 \\ & \cdot \\ & \cdot \\ & \cdot \\ & 0 \\ \frac{1}{2}\beta_m^{n+1} & u_{m+1}^{n+1} \end{bmatrix} + \begin{bmatrix} \frac{1}{2}\beta_1^{n} & u_o^{n} \\ & 0 \\ & \cdot \\ & \cdot \\ & \cdot \\ & 0 \\ \frac{1}{2}\beta_m^{n} & u_{m+1}^{n} \end{bmatrix}
$$

where

$$X_m = 1 - ka_m^{n+1}/2 + [k/(\delta z)^2]b_m^{n+1}$$

$$Y_m = 1 + ka_m^n/2 - [k/(\delta z)^2]b_m^n$$

$$\beta_m^n = [k/(\delta z)^2]b_m^n.$$

The nonzero elements of the last two vectors represent quantities to be determined by boundary conditions at the top and bottom of the troposphere model. The first mesh point of the model was taken to be one-half mesh unit above the ocean surface. The zeroth mesh point was therefore one-half mesh unit below the surface. The ocean was assumed to be a perfect conductor. Therefore, the bottom boundary condition was $u_o^n = u_1^n$.

The boundary condition at the top should be that only outgoing waves are present. This is modeled by placing an absorber in the upper part of the profile. Following Brock [1978], the upper one-fourth of the profile was modified by

$$n^2 \leftarrow n^2 - i0.01 \ \exp \ \{-[(z - z_{top})/(z_{top}/4)]^2\}$$

where i is $\sqrt{-1}$ and the difference in sign is the result of the electromagnetic convention of waves traveling as $\exp(i\omega t)$ compared to the acoustical convention of waves traveling as $\exp(-i\omega t)$

The value of u_{m+1}^n was determined from the condition

$$\frac{1}{u}\frac{du}{dz} = -ik(2\frac{dM}{dz} \times 10^{-6}z)^{1/2} = G$$

or

$$u_{m+1} = u_m\left(\frac{1}{\delta z}+\frac{G}{2}\right)\Big/\left(\frac{1}{\delta z}-\frac{G}{2}\right)$$

where

$$M = (n-1+z/a) \times 10^6$$

a is the radius of the Earth, and δz is the vertical mesh size.

Another upper-boundary condition was tried for which the u_{m+1} value was taken to be its value as determined by ray tracing. In such cases, no absorber was used. No such run was successful, however.

III. STARTING SOLUTION

The parabolic equation method requires a set of starting values as a function of height. Two methods were employed. One was to use a "free space" condition except that the Earth as a flat perfect conductor was included. It was taken to be valid only somewhat close to the transmitter and in practice was used at 2.5 km to 10 km from it. It is given by

$$u = [\exp(\alpha_1) - \exp(\alpha_2)]/\sqrt{r_o}$$

where

$$\alpha_1 = -ik\{[r_o^2 + (z-z_T)^2]^{1/2} - r_o\}$$

$$\alpha_2 = -ik\{[r_o^2 + (z+z_T)^2]^{1/2} - r_o\}$$

z_T is the height of the transmitter, and r_o is the range.

The other method was a ray-trace formulation developed by R. A. Pappert. It was used at a distance of 4 km from the transmitter.

IV. EXAMPLES

Three examples are given of the implicit-finite-difference method. The two models that were successfully used were each of a surface duct.

The first model was of a bilinear profile, such that the upper part had a slope of dM/dz = 0.118 and the lower part had a slope of dM/dz = -0.118. The apex was at 14 m, the transmitter was at 25 m, and the receiver at 5 m. Starting values were found at 4 km by using the ray-trace formulation. The vertical mesh size was 0.25 m and the horizontal step size was 100 m. An absorber was used from 96 through 128 m. Path loss as a function of distance is shown in figure 1 as the solid line. The x symbols in the figure are waveguide normal mode theory values as found in the program MLAYER. It is seen that the values agree very closely.

The second example is of a multilinear profile representing a 14-m surface duct [from Hitney, 1988]. The profile is given in table 1. The transmitter was at 25 m and the receiver at 5 m. The "free-space" starter was used at 2.5 km from the transmitter. The vertical mesh size was 0.25 m and the horizontal step size was 20 m. An absorber was used from 75 through 100 m. Path loss as a function of distance is shown in figure 2. The x symbols are waveguide normal mode values. It is seen that the values agree very closely.

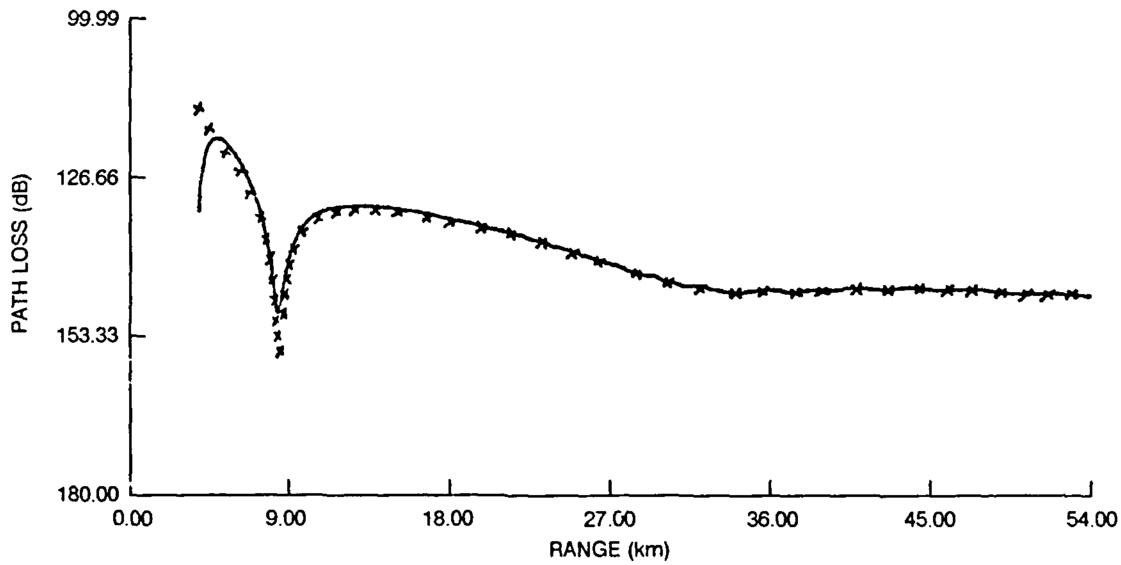

Figure 1. Bilinear model.

Table 1. 14-m evaporation duct.

Height (m)	M-units
0.000	0.000
0.040	-3.900
0.100	-5.340
0.200	-6.400
0.398	-7.460
0.794	-8.490
1.585	-9.470
3.162	-10.350
6.310	-11.040
12.589	-11.320
14.000	-11.330
25.119	-10.870
39.811	-9.750
50.119	-8.820
63.096	-7.560
79.433	-5.880
100.000	-3.670

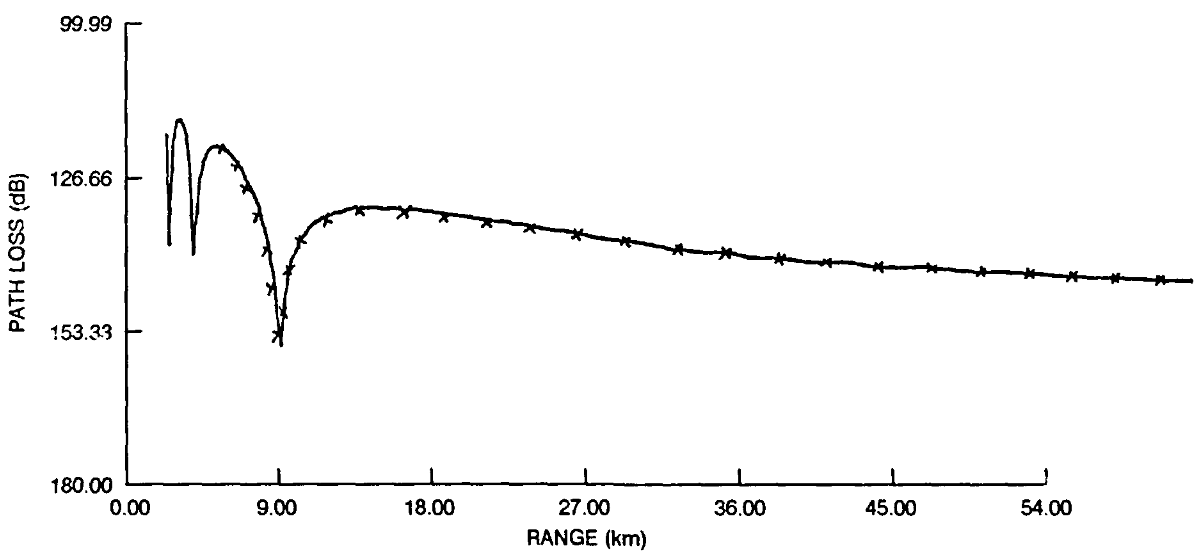

Figure 2. Multilayer 14-m duct.

A standard atmosphere model also was tried (figure 3). The "free-space" starter was used at 10 km. The vertical mesh size was 1 m and the horizontal step size was 20 m. In the first 35 km, the resulting values are the same as found by MLAYER. Beyond that, the path loss values should have been very high ones, i.e., signal level values should have been very low. Instead, a relatively low level of noisy path loss values resulted. A vertical mesh size of 0.25 m was also tried with much the same result. Apparently, the implicit-finite-difference method is not effective in obtaining very low signal levels at 9.6 GHz.

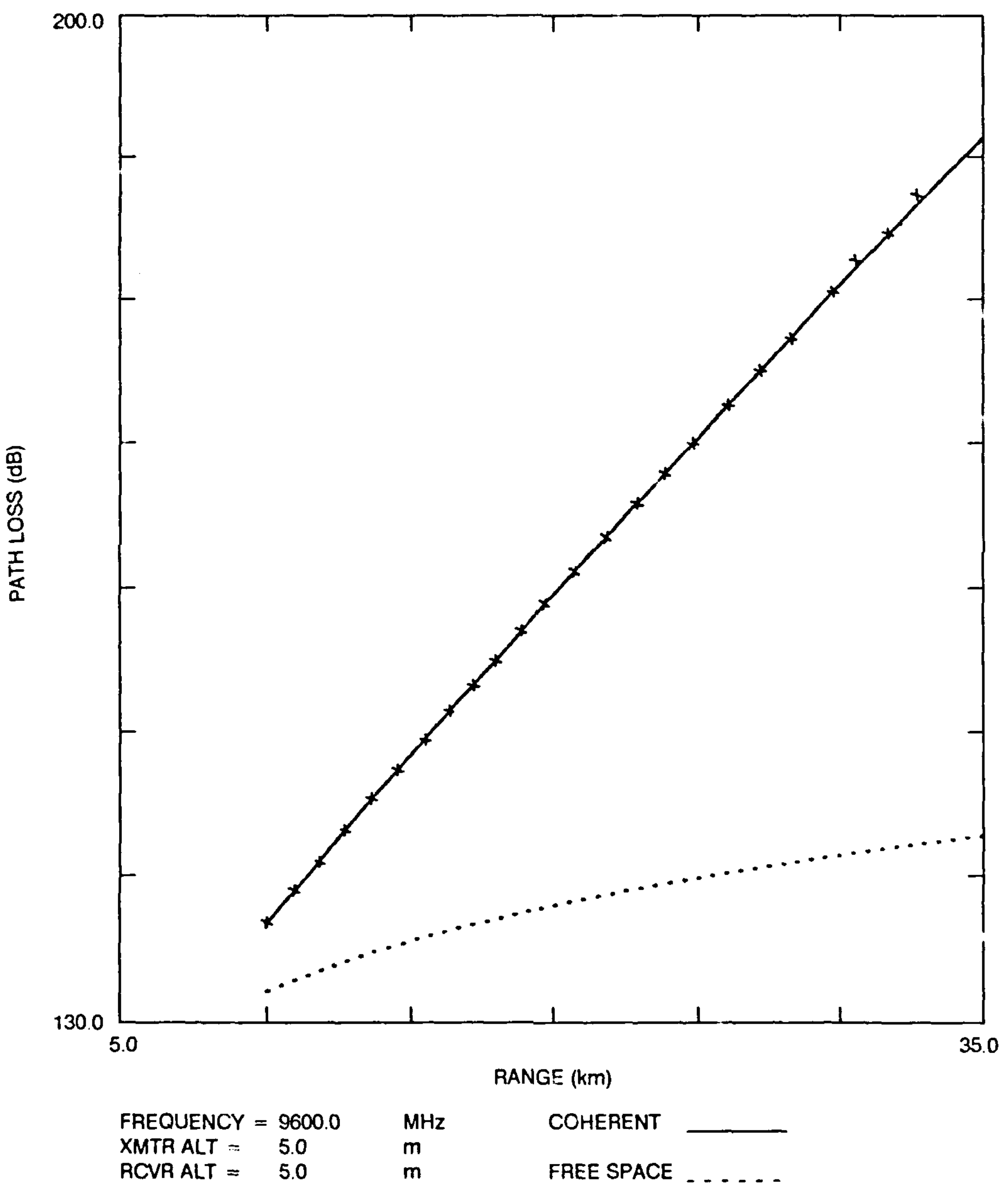

Figure 3. Standard atmosphere.

V. SUMMARY AND CONCLUSIONS

The theory of the implicit-finite-difference method has been summarized and the boundary conditions described. Examples indicate that, for sufficiently high signal levels, the IFD method is adequate at 9.6 GHz. However, the method did not give correct results beyond 35 km for the standard atmosphere case in which signal levels are very low.

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C. H. Shellman

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